**General Equation of Second Degree**

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## Conic Section

If a point moves in a place in such a way that the ratio of its distances from a fixed point in the plane, to its perpendicular distance from a fixed line is always constant, the locus of the point is called a conic section.

The fixed point is called the focus and the fixed line is called the directrix. The constant ratio is called the eccentricity of the conic, i.e. .

If , then and the conic is a parabola.

If , then and the conic is an ellipse.

If , then and the conic is a hyperbola.

The general equation of second degree, , represents a conic (i.e. pair of straight line, parabola, ellipse, circle or hyperbola) under the following cases:

Case 1: Straight Lines

()

i) Lines are parallel

ii) Lines are perpendicular

Case 2: Circle

Case 3: Parabola

Case 4: Ellipse

Case 5: Hyperbola

i) Rectangular Hyperbola

Example:

Find the type of conic section represented by the equation:

- (i)

Comparing (i) with ,

, , , , ,

Since and , the equation represents a hyperbola.

## Three-Dimensional Geometry (3D)

### Direction Cosine (d’cs)

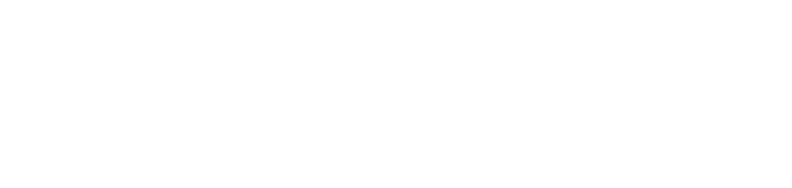
If a line makes an angle , and with the positive directions of the , and axes respectively, then their cosine values, , , are called the direction cosines of the line.

Generally, they are denoted by , and , i.e. , and .

(This is proven later.)

### Direction Ratios (d’rs)

If any three numbers , and , which are proportional to the d’cs , and , then the numbers are called direction ratios. (Basically, vector directions.)



d'rs are , , (no brackets)

d’cs are and

Find the direction angles of a line joining the points and .

The d’rs of a line joining the two points are , , , i.e. , , .

Let be any line where is the origin, is any point that makes angles of , and with , , and axes respectively. Let be the length of the line .

From ,

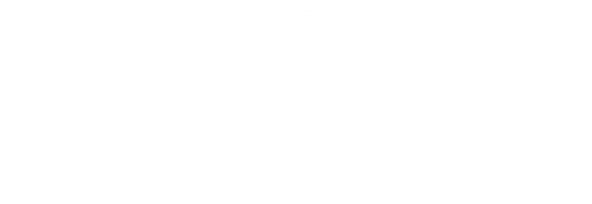
Similarly,

Test if the numbers , and are d’rs or d’cs.

These are direction cosines (d’cs).

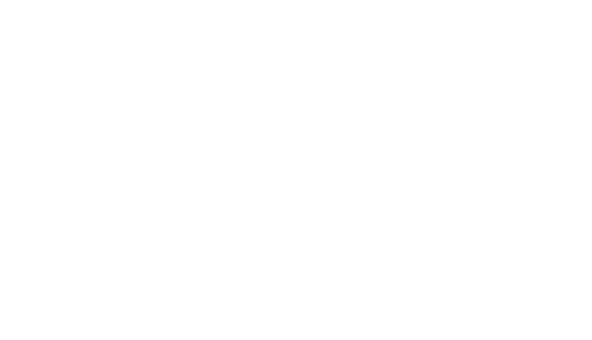
## Projection

### Projection of a Point on a Line



This is the point that is the foot of the perpendicular from to .

### Projection of a Segment of a Line on another Line



Project of a segment of a line joining the points and on another line whose d’rs are , , .

The following 3 questions have the same solution:

1. Find the angle between two lines.

2. If and are the direction cosines (d’cs) of any two lines and is the angle between them, then show that .

3. If and are the direction ratios (d’rs) of any two lines and is the angle between them, then show that .

Answer:

Let and be any two lines where is the origin, has coordinates and has coordinates .

Let be the angle between and .

The projection of on is .

The projection of on is also

Case 1: or

Case 2: (parallel) d’cs same

d’rs proportional